

The triangle $\triangle PQD$ is isosceles ($|DP| = |DQ|$ due to being tangents), therefore we have $\angle DQP = \angle QPD$ on the other hand $\angle DQP = \beta + \angle QSP$ (outer angle of triangle $\triangle PQR$). Considering $\angle QPD = \alpha + \angle RPD$, we would have $\alpha = \beta$ if $\angle QSP = \angle RPD$. To show $\angle QSP = \angle RPD$, we prove that the triangles $\triangle DPR$ and $\triangle DPS$ are similar.

Note: 2 triangles are similar if they have 1 angle of the same measure and the ratio of the enclosing triangles sides is equal.

Both triangles share the angle $\angle RDP$ and the ratio of the enclosing triangles sides is $\frac{|DP|}{|DS|}$ and $\frac{|DR|}{|DP|}$.

The tangent-secant theorem applied on the outer circle yields $|DP|^2 = |DR| \cdot |DS|$, therefore $\frac{|DP|}{|DS|} = \frac{|DR|}{|DP|}$. Together we have equal ratios and an angle of equal measure, hence the triangles $\triangle DPR$ and $\triangle DPS$ are similar.

