

Since since the set we are minimizing over is a curve, a parametrization will reduce the problem to finding the minimum of function of one variable. The circle equation  $x^2 + y^2 = 1$  of the cylinder can be parametrized by  $x(t) = \cos(t)$  and  $y(t) = \sin(t)$ , due to  $z = 1 - x + y$  we have  $z(t) = 1 - x(t) + y(t)$  and hence a complete parametrization. So minimizing  $f(x, y, z) = x + 2y + 3z$  over the curve is equivalent to minimizing  $f(t) = f(x(t), y(t), z(t)) = \cos(t) + 2\sin(t) + 3 \cdot (1 - \cos(t) + \sin(t)) = 3 - 2\cos(t) + 5\sin(t)$  over  $[0, 2\pi]$

With  $f'(t) = 2\sin(t) + 5\cos(t)$  and  $f'(t) = 0$  we get  $\tan(t) = \frac{\sin(t)}{\cos(t)} = -\frac{5}{2}$  and  $t = k \cdot \pi - \arctan\left(\frac{5}{2}\right)$ . This yields us 2 candidates for a minimum in  $[0, 2\pi]$  :  $\{\pi - \arctan\left(\frac{5}{2}\right), 2\pi - \arctan\left(\frac{5}{2}\right)\}$ .

We verify the minimum using  $f''(t) = 2\cos(t) - 5\sin(t)$  . :

$$f''\left(\pi - \arctan\left(\frac{5}{2}\right)\right) \approx f''(1.9513027042) = -5.385164807 < 0$$

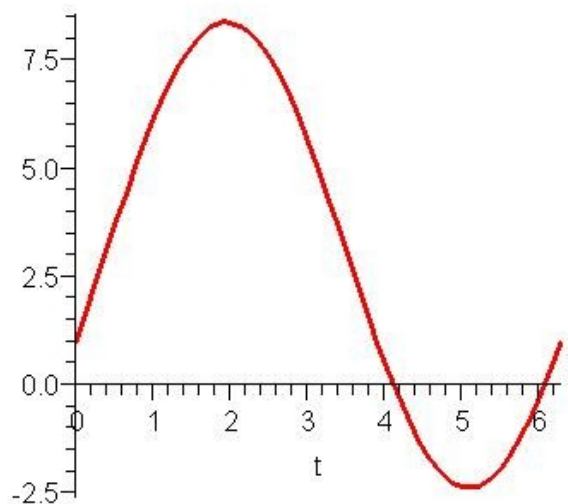
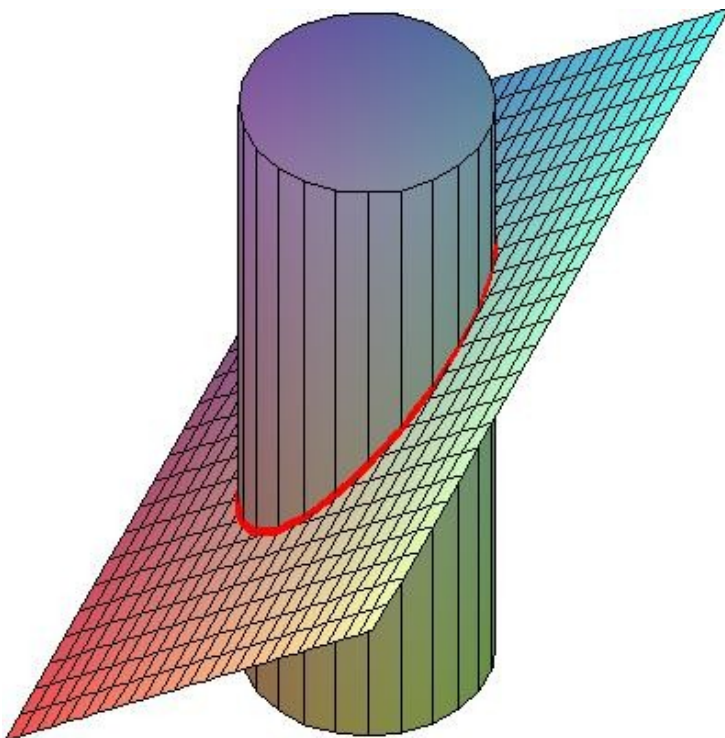
$$f''\left(2\pi - \arctan\left(\frac{5}{2}\right)\right) \approx f''(5.092895358) = 5.385164807 > 0$$

So  $f(t)$  has a minimum at  $t = 2\pi - \arctan\left(\frac{5}{2}\right)$ , which means the curve has a minimum at

$$\left(x\left(2\pi - \arctan\left(\frac{5}{2}\right)\right), y\left(2\pi - \arctan\left(\frac{5}{2}\right)\right), z\left(2\pi - \arctan\left(\frac{5}{2}\right)\right)\right) = \left(\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}, \frac{\sqrt{29}-7}{\sqrt{29}}\right) \approx (0.371, -0.928, -0.300)$$
 and its

$$\text{value is } f\left(2\pi - \arctan\left(\frac{5}{2}\right)\right) = 3 - \sqrt{29} \approx -2.385$$

The following graphics illustrate the situation:



— f(t)