Since since the set we are minimizing over is a curve, a parametrization will reduce the problem to finding the minimum of function of one variable. The circle equation $x^2 + y^2 = 1$ of the cylinder can be parametrized by $x(t) = \cos(t)$ and $y(t) = \sin(t)$, due to $z = 1 - x + y$ we have $z(t) = 1 - x(t) + y(t)$ and hence a complete parametrization. So minimizing $f(x, y, z) = x + 2y + 3z$ over the curve is equivalent to minimizing $f(t) = f(x(t), y(t), z(t)) = \cos(t) + 2\sin(t) + 3\cdot(1 - \cos(t) + \sin(t)) = 3 - 2\cos(t) + 5\sin(t)$ over $[0,2\pi]$

With $f'(t) = 2\sin(t) + 5\cos(t)$ and $f'(t) = 0$ we get $\tan(t) = \dfrac{\sin(t)}{\cos(t)} = -\dfrac{5}{2}$ and $t = k\cdot\pi - \arctan\left(\dfrac{5}{2}\right)$.This yields us 2 candidates for a minimum in $[0,2\pi]$ : $\{\pi - \arctan\left(\dfrac{5}{2}\right), 2\pi - \arctan\left(\dfrac{5}{2}\right)\}$.

We verify the minimum using $f''(t) = 2\cos(t) - 5\sin(t)$. :

$$f''(\pi - \arctan\left(\frac{5}{2}\right)) \approx f''(1.9513027042) = -5.385164807 < 0$$

$$f''(2\pi - \arctan\left(\frac{5}{2}\right)) \approx f''(5.092895358) = 5.385164807 > 0$$

So $f(t)$ has a minimum at $t = 2\pi - \arctan\left(\dfrac{5}{2}\right)$, which means the curve has a minimum at

$(x(2\pi - \arctan\left(\frac{5}{2}\right)), y(2\pi - \arctan\left(\frac{5}{2}\right)), z(2\pi - \arctan\left(\frac{5}{2}\right))) = (\frac{2}{\sqrt{29}}, \frac{-5}{\sqrt{29}}, \frac{\sqrt{29}-7}{\sqrt{29}}) \approx (0.371, -0.928, -0.300)$ and ist value is $f(2\pi - \arctan\left(\frac{5}{2}\right)) = 3 - \sqrt{29} \approx -2.385$

The following graphics illustrate the situation: