Consider that the sum of digits modulo 9 is equal to the number modulo 9, hence by constructions all elements of sequence are equal modulo 9.

Now we have $2^6 \equiv 64 \equiv 1 \bmod 9$ and $2006 = 334 \cdot 6 + 2$, which yields us:

$2^{2006} = (2^6)^{334} \cdot 2^2 \equiv 1^{334} \cdot 4 = 4 \bmod 9$. In other words the constant limit digit is 4.