We need to minimize *f* subject to the two constraints $g(x,y,z) = x - y + z - 1 = 0$ and $h(x,y,z) = x^2 + y^2 - 1 = 0$

As we have two constraints we use two Lagrangian multipliers λ and μ with the Lagrangian function:

$L(x,y,z\lambda,\mu) = f(x,y,z) + \lambda g(x,y,z) + \mu h(x,y,z)$

So $\nabla L = ((1+\lambda+2x\mu),(2-\lambda+2y\mu),(3+\lambda),(x-y+z-1),(x^2+y^2-1))$

Letting $\nabla L = 0$ we get $x = 1/\mu$ and $y = -5/2\mu$

Substituting these values into $x^2 + y^2 = 1$ we get $1/\mu^2 + 25/4\mu^2 = 1$ so $\mu = \pm\sqrt{29}/2$

Hence $x = \mp 2/\sqrt{29}$ and $y = \mp 5/\sqrt{29}$

Now as $x - y + z = 1$ then $z = 1 \pm 7/\sqrt{29}$

This gives two points $\left(-2/\sqrt{29}, -5/\sqrt{29}, 1 + 7/\sqrt{29}\right)$ and $\left(2/\sqrt{29}, 5/\sqrt{29}, 1 - 7/\sqrt{29}\right)$ at which *f* takes the values $3+\sqrt{29}$ and $3-\sqrt{29}$ respectively.

Hence the minimum value $3-\sqrt{29}$ occurs at $\left(2/\sqrt{29}, 5/\sqrt{29}, 1 - 7/\sqrt{29}\right)$.